The evaluation of duration: a literature review

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Abstract: For public investors, most of them will choose to invest bonds since they are low-risk investment choices. Therefore, it is of great importance to analyze the risks of bonds, and duration is one of the convenient strategies. Based on the studies and research made by different people, this paper summarizes the advantages and disadvantages for duration strategy to assess interest rate risks of a bond primarily. In the disadvantage part, there important assumptions in duration model are listed. In addition, due to these disadvantages, author makes a comparison on duration, effective duration, modified duration and convexity, in terms of which disadvantages of duration they have solved. Besides, this paper will discuss influencing factors that affect the approximation of duration model, such as default and call risks, and non-parallel movement of the yield curve. In the last part, author briefly concludes opinions discussed about in previous parts, then put forward some individual thoughts and perspectives.

1. Introduction

1.1 Background

It is universally acknowledged that bonds are the foundation of the world's debt capital markets, which are, in turn, the bedrock of the global economy [1]. For public investors, many of them will choose to invest bonds since they are low-risk investment choices. Even though it is low risk, there still exists different kinds of risks, such as interest rate risks. Therefore, for investors, they are desired to analyze or mitigate the risks of bonds.

According to large amount of research and experiences, single-factor duration models (which assume that changes in interest rates for all maturities are completely connected) are beneficial in practice. The research suggests that the duration model measure of duration works quite well in contrast to its more complex counterparts and looks to be cost-effective due to its simplicity [2].

Nevertheless, it is inevitable for duration model to have errors and inaccuracies, since the duration model is just a simple estimation model related to only one variable. In this case, great significance has been attached to the evaluation and improvement of duration model. Through great effort and much research, several limitations have been discovered and improved models based on duration such as modified duration, convexity and effective duration have been created.

1.2 Nessecity

Although several flaws have been uncovered, and improved models have been proposed, thanks to a lot of hard work and study, a comprehensive conclusion or report on the researchers' opinions still could not be found. Therefore, the author would like to make a conclusion after reading articles and journals written by different researchers.

1.3 Objective

Based on different opinions put forward by different people, author tries to make a conclusion on the evaluation of duration model. Although duration model is widely used to estimate the change of bond price, in terms of the change in interest rate, yields have altered under different circumstances in

ways that are able to be fully articulated by simply saying "yields went up" or "yields went down." Therefore, this paper will find out the factors affecting the estimation of bond duration [3].

1.4 Contribution

After reading various viewpoints on the evaluation of the duration model expressed by different researchers, the author attempts to reach a judgement on them, dividing into three parts: the advantages and disadvantages of duration model, a comparison between duration model and effective duration and factors affecting a bond's duration. Besides, author might put forward some innovative opinions on the evaluation of duration model or a new perspective to study the duration.

1.5 Remaining Structure

In the "The fundamental research about duration model" part, author will discuss about the advantages and disadvantages of duration model.

In the "The comparison between research on duration and research on other models" part, the limitations of duration will lead to the ways to improve the limitations, making a comparison between duration and other models.

In the "The influencing factors of bond duration" part, this paper will talk about some factors that might affect the duration of a bond.

In the last "Conclusion" part, the author will conclude what has been discussed in the whole paper and try to make an individual innovation on the evaluation of duration model.

2. The fundamental research about duration model

2.1 Advantage

2.1.1 A simple summary statistic of the effective average maturity

According to Frank and Rupinder [4], the duration of a bond is considered to be a better method to characterize bond flow time than maturity to some extent. The duration of a bond is computing the weighted average of the maturity of individual cash flows. In fact, the weight assigned to each cash flow should be proportional to the "importance" of that payment [5] to the value of a bond. In other words, the duration of a bond summarizes the performance of the overall bond, which from the investors' perspective, could also be thought as the expected waiting years for an investor to receive all the cash flow benefits from a bond. Therefore, compared to a bond's maturity, a bond's duration could better reflect the interest rate risk investors need to take, due to the expected waiting years the duration computes, when investors want to compare different bonds.

2.1.2 Duration and portfolio immunization

Another advantage is that portfolio immunization could be achieved by applying duration model. Setting the portfolio's derived time characteristic to the length of the planning period [6], We are able to figure out the optimal weighting of different bonds in a portfolio respectively, thus achieving immunizing risks of the portfolio. According to Fisher and Weil's test [7], the immunization using duration was better than that using maturity around 75% of time of the test, since duration strategy could produce consistently lower standard deviations for the difference between realized and promised returns, compared to maturity strategy.

Besides, if the knowledge of duration and portfolio immunization is applied into a firm, Samuelson [8] and Grove [9] concluded that by having the weighted duration of its assets equal to the weighted duration of its liabilities, the firm could immunize its interest rate risks.

2.2 Disadvantage

2.2.1 Assumption 1: The price yield curve is a linear function [10].

An assumption in duration model is that the relationship between price and yield is linear. Whereas, this assumption could not be realized in real world, and the price yield curve should be a convex curve. In this case, for a big change in the yield, duration model will produce a big error. In other words, the

duration model is becoming less and less accurate, as the degree of yield variation increases [11]. Moreover, since the duration model assumes the price yield curve as a linear function, which is always under the exact convex curve, the situation, that how much bond prices would rise when yields fall is underestimated and how much they would fall when yields rise is overestimated, might occur.

2.2.2 Assumption 2: The future cash flows of bonds will not change.

There is another assumption that the cash flows of bonds do not change as interest rates fluctuate [12], because the duration formula is using the future cash flows of a bond. However, this assumption might not make sense to bonds whose future cash flows are uncertain, like floating rate bonds and callable bonds. For instance, if duration model is used to analyze a callable bond, it is possible that the issuer would call the bond back and issue another bond at a new interest rate, if interest rate fell to a low degree. In this case, a big error will be produced by duration model, which has a negative impact on the analysis of interest rate risks.

2.2.3 Assumption 3: The interest rates can only change by a parallel shift.

The last point is that duration model supposes that when interest rates change, the yield curve only moves in parallel. It is the case only if for all maturities, interest rates fluctuate by the same amount [13]. Nevertheless, this condition could be hardly satisfied, since the price yield curve not only moves in parallel, but also could change its slope or its convexity.

3. The comparison between research on duration and research on other models

3.1 Duration model and effective duration

As for the differences between duration and effective duration, it could be obviously seen from their formulas. Compared to the duration, which is computing the weight of each cash flow of the whole bond price, effective duration uses the change in bond price, in response to the change in interest rate. You could see from the first row in table 1 below, the first difference between duration and effective duration according to their formulas is that duration is related to the change of yield, while effective duration is directly subject to the interest rate change. Because of some bonds have embedded options [14] like calling back bonds early, which is no longer related to the yield of maturity, effective duration can be used to analyze more types of bonds than duration, and it can more intuitively reflect the relationship between price changes and interest rate changes.

In addition, the second difference between duration and effective duration is that duration is computing the changes of all cash flows, while effective duration directly computes the bond price change. Therefore, if investors would like to analyze the risks of bonds whose future cash flows are uncertain like callable corporate bonds, effective duration should be more useful and accurate than duration, which is showed in table 1 as well. According to Sarkar [15], callable corporate bonds are exposed to default risk and call risk, because either default risk or call risk will obviously modify the timing and quantity of cash flows, and hence alter the sensitivity of bond value to interest rate changes. To analyze this kind of bonds, effective duration should be applied instead of duration, since their cash flows become unknown.

The table below illustrates a comparison between duration model and effective duration, in terms of their dependent variables, attributes and their own applications.

Table 1. The Comparison Between Duration and Effective Duration

	Dependent variable	Attribute	Application
Duration	the changes of all cash flows	the change of yield	bonds with fixed cash flows
Effective duration	the change of bond price	the change of interest rate	bonds with uncertain cash flows

3.2 Duration model and modified duration, convexity

It is widely known that duration is a function of curvilinear bond price yield relationship, which measures the slope value of the price-yield curve at a given change of the yield-to-maturity. On the contrary, with respect to the modified duration, it could be seen as the first derivative [16] of the price yield curve, which could calculate the slope of the curve at each point. As a consequence, modified duration is thought to be able to reflect the sensitivity of bonds more dynamically. Besides, since the figure that the percentage change of bond price to the percentage change of interest rate could be known through modified duration strategy, investors are enabled to know more precisely about the impact of interest rate changes on bond prices and the specific value of percentage changes in bond prices under all interest rates respectively.

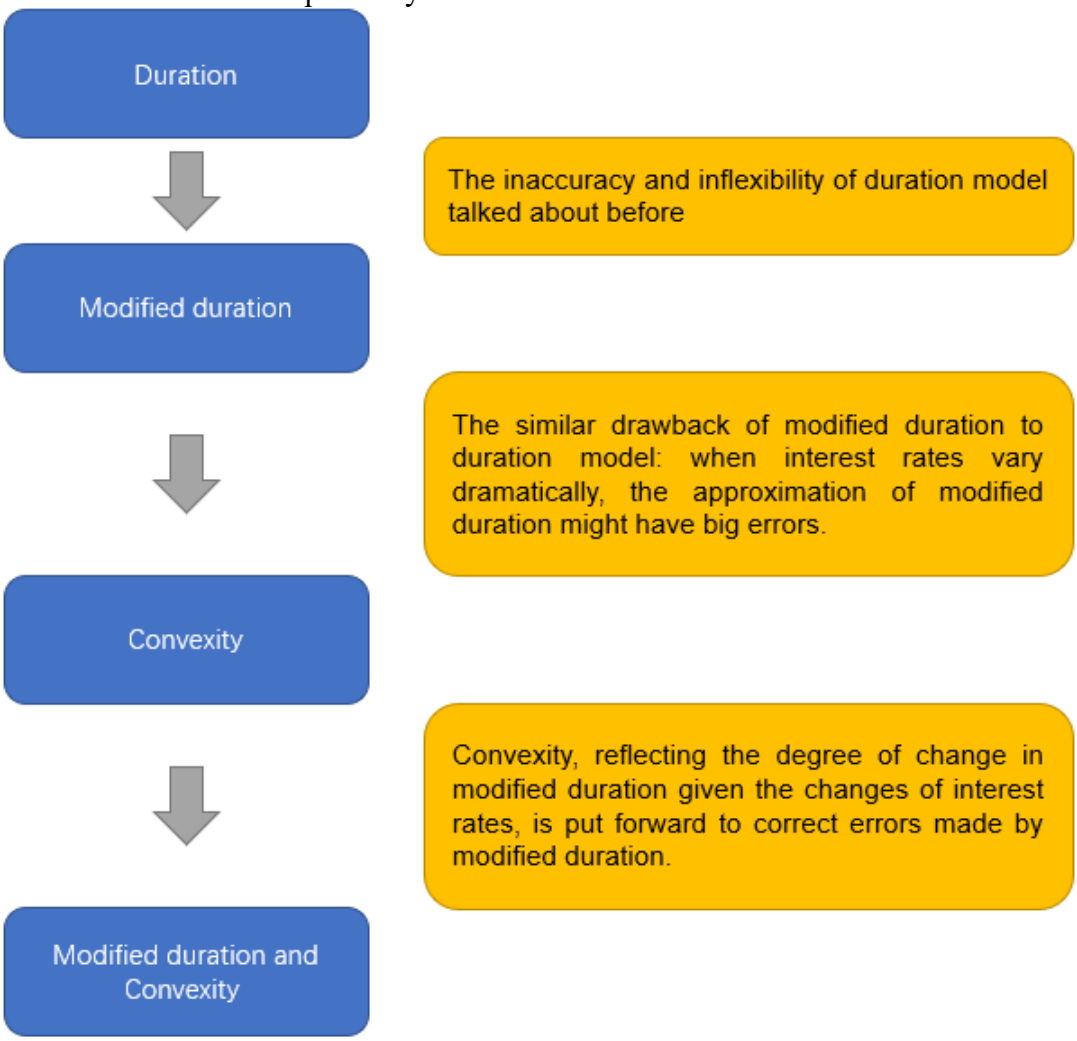

Figure 1. The Improvement Process of Duration Model

Eventually, although modified duration resolves one limitation of duration model, which is that it could not assess the sensitivity of bond prices to interest rates, the disadvantage of modified duration might be similar to that of duration to some extent, that is its inaccuracy showed in figure 1. Under such circumstances, the convexity strategy has been put forward. In fact, convexity is the price yield curve's second derivative, which represents the degree of change in modified duration given the changes of interest rates.

According to C. Steven Cole and Philip J. Young [16], most existing studies ignore bond price volatility. It means that there is the possibility that a bond could fluctuate significantly or slightly, which could have a big impact on the estimation of interest rate risk. For example, when interest rates vary slightly, modified duration is able to get very precise conclusions. Whereas, when interest rates vary dramatically, the approximation of modified duration might have big errors. In this case, convexity is needed in order to correct the errors made by modified duration. Therefore, equipped with

both modified duration and convexity, the estimated value at risks could be more accurate, regardless of the degree of the interest rate changes.

The figure below completely displays the long-term optimization process of duration model, which enables readers to know how did the duration model develop.

4. The influencing factors of bond duration

4.1 Default and call risk on bond duration

Primarily, if duration was used to measure bonds in the presence of default risk, the presence of default risk would affect the evaluation of duration a lot. Nawalkha built on previous research by looking at how company asset prices are affected by interest rates and found that the default risk-adjusted durations of defaultable zero-coupon bonds might be less, equal to, or greater than the Macaulay durations, according to his research [17]. Therefore, it is found that compared to those default-free bonds, the durations of defaultable bonds might be longer or shorter, relying on the relationship between default intensity and interest rates [18].

Additionally, Fons calculated the empirical durations of corporate bond indices and showed that, from 1980 to 1988, the durations of corporate bond indices were always fewer than the Macaulay durations, with the gap between the two measures widening as bond ratings decrease [19]. Moreover, according to Duffee's experiment [20], it is suggested that risk affects bond duration. In the presence of call features, since issuers have the rights to redeem bonds before the bonds get mature. As a result, the weighted average time to maturity calculated, which is duration, is always shortened.

4.2 Non-parallel movement of the yield curve

Even though duration model assumes that the yield curve simply moves in parallel, it could hardly be the case as a matter of fact. The movement could be decomposed into three parts, which are parallel movements, slope changes and convexity changes. In duration model, it is assumed that the interest rates of bonds would change accordingly for the whole maturity time. Nevertheless, at any point in time or over a short period of time, a change in interest rates might cause a change in the slope or a change in the convexity of the yield curve, influencing the estimation of bonds' duration. For a small range of interest rate changes or short-term bonds that do not typically parallel rate increases [21], key rate duration should be utilized to estimate a small period, since it signals projected price changes when there are adjustments in the yield curve that are not parallel across all maturities.

5. Conclusions

Generally speaking, the advantages and disadvantages of duration model have been almost fully explored for a large period of time. Moreover, anchored in those limitations of duration model, a large number of improved models have been put forward, like modified duration and key rate duration. However, there is still not a combined model which could solve more than one limitations of duration models. If such model was created, it would be quite more popular than any other models.

References

[1] Choudhry M. An introduction to bond markets[M]. J. Wiley & Sons (Asia), 2006.

[2] G.O. Bierwag, G. G. Kaufman & A. Toevs, Duration: Its Development and Use in Bond Portfolio Management, Financial Analysts Journal, Volume 39, 1983, 31/12/2018.

[3] R. Litterman & J. Scheinkman, Common Factors Affecting Bond Returns, June 1991.

[4] F. K. Reilly & R. S. Sidhu, The Many Uses of Bond Duration, Financial Analysts Journal, 1980, Vol. 36, No. 4, pp. 58-72.

[5] Bodie Z, Kane A, Marcus A J. Essentials of investments[J]. Irwin/McGraw Hill, 2017.

[6] G. O. Bierwag, George G. Kaufman and C. Khang, Duration and Bond Portfolio Analysis: An Overview, The Journal of Financial and Quantitative Analysis, Nov., 1978, Vol. 13, No. 4, pp. 671-681

[7] G. O. Bierwag, George G. Kaufman and C. Khang, Duration and Bond Portfolio Analysis: An Overview, The Journal of Financial and Quantitative Analysis, Nov., 1978, Vol. 13, No. 4, pp. 671-681

[8] Samuelson, P. A. "The Effect of Interest Rate Increases on the Banking American Economic Review (March 1945), pp. 16-27.

[9] Grove, M. A. "A Model of the Maturity Profile of the Balanced Sheet." Metroeconomi- ca (April 1966), pp. 40-55.

[10] Available from: https://baike.eastmoney.com/item/%E9%BA%A6%E8%80%83%E5%88%A9%E4%B9%85%E6%9C%9F

[11] Available from: http://www.haoruiedu.com/3g/news_show_368_1_4.html

[12] Available from: https://baike.eastmoney.com/item/%E9%BA%A6%E8%80%83%E5%88%A9%E4%B9%85%E6%9C%9F

[13] H. Gifford Fong & Oldrich A. Vasice, A Risk Minimizing Strategy for Portfolio Immunization, The Journal of Finance, No. 5, 1984.

[14] Z. Bodie, A. Kane and A. J. Marcus, Investments, the MaCraw-Hill company, ninth edition

[15] S. Sarkar, Effective duration of callable corporate bonds: Theory and evidence, Michael DeGroote School of Business, McMaster University, 1280 Main Street West, Hamilton, ON L8S 4M4, Canada.

[16] C. Steven Cole and P. J. Young, Modified Duration and Convexity with Semiannual Compounding, Journal of Economics, Vol. 19, 1995, pp. 1-15.

[17] S. K. Nawalkha, A Contingent Claims Analysis of the Interest-Rate-Risk Characteristics of Corporate Liabilities, Journal of Banking and Finance, 20(1996), pp. 227-245.

[18] Y.A. Xie, S. Liu and C. Wu, Duration, default risk, and the term structure of interest rates, Journal of Financial Research, 28 (2005), pp. 539-554.

[19] J.S. Fons, Default risk and duration analysis, E.I. Altman (Ed.), The High Yield Debt Market: Investment Performance and Economic Impact, Dow Jones-Irwin, Homewood, 1990

[20] G.R. Duffee, The relation between Treasury yields and corporate bond yield spreads, Journal of Finance, 53 (1998), pp. 2225-2241

[21] Available from: https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/key-rate-duration/